

ON SOME GRAVITOMAGNETIC SPIN-SPIN EFFECTS FOR ASTRONOMICAL BODIES

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In this paper we look at the gravitational spin-spin interaction between macroscopic astronomical bodies. In particular, we calculate their post-Newtonian orbital effects of order $\mathcal{O}(c^{-2})$ on the trajectory of a spinning particle with proper angular momentum \mathbf{s} moving in the external gravitomagnetic field generated by a central spinning mass with proper angular momentum \mathbf{J} . It turns out that, at order $\mathcal{O}(e)$ in the orbiter's eccentricity, the eccentricity the pericenter and the mean anomaly rates of the moving particle are affected by long-term harmonic effects. If, on one hand, they are undetectable in the Solar System, on the other, maybe that in an astrophysical context like that of the binary millisecond pulsars there will be some hopes of measuring them in the future.

1. Introduction

In recent years the topic of the general relativistic post-Newtonian effects of order $\mathcal{O}(c^{-2})$ on various features of the motion of spinning particles freely orbiting a central astronomical body has received great attention both theoretically and experimentally. More precisely, when a spinning particle moves in an external gravitational field one has to describe both its spin precession and the influence of the spin on its trajectory¹.

The geodetic, or De Sitter, precession² refers to the coupling of the static, gravitoelectric part of the gravitational field due to the Schwarzschild metric generated by a central, non-rotating object to the spin of a particle freely orbiting around it. It has been measured for the Earth-Moon orbit, thought of as a giant gyroscope, in the gravitational field of the Sun with 1% accuracy³. It should be measured for four superconducting gyroscopes in the gravitational field of the Earth by the important GP-B mission⁴ at a claimed relative accuracy level of 2×10^{-5} . Finally, it might be possible to detect it also in some binary pulsar systems⁵.

The Lense-Thirring drag of the inertial frames⁶ is an effect due to the stationary, gravitomagnetic part of the gravitational field of a rotating central mass with proper angular momentum \mathbf{J} on the geodesic path of a freely falling test particle, i.e. considered to be not spinning itself^a. In 1998 the first evidence of the Lense-

^aSuch effect could be thought of as a spin-orbit interaction between the spin of the central object and the orbital angular momentum of the test body. For some spin-orbit effects induced by the

Thirring effect in the gravitational field of the Earth has been reported⁸ with a claimed accuracy of almost 20%. It is based on the analysis of the laser-ranging data of the LAGEOS and LAGEOS II geodetic satellites. The launch of the proposed LAGEOS-like LARES satellite⁹ could allow to measure such effect with an accuracy probably better than 1%. Another interesting gravitomagnetic effect of order $\mathcal{O}(c^{-2})$ on the orbit of a test particle has been recently derived in reference¹⁰; it is due to the temporal variability of the Earth's angular momentum. Unfortunately, it is too small to be detected with Satellite Laser Ranging.

The spin of the central object affects also the spin of a particle freely orbiting it in a way discovered by Schiff¹¹ in 1959. The detection of this subtle precessional effect, in addition to the geodetic precession, is one of the most important goals of the GP-B mission⁴; the claimed accuracy amounts to 1%.

In this paper we are interested in looking for some orbital effects due to the spin-spin gravitational coupling on the geodesic path of a spinning extended particle with mass m and proper angular momentum \mathbf{s} freely orbiting around a central body of mass M and proper angular momentum \mathbf{J} . The gravitational spin-spin coupling in the quantum mechanical domain has been treated in references^{12, 13}.

The paper is organized as follows. In Section 2 we derive the gravitational Stern–Gerlach force with some simplifying assumptions. In Section 3 we work out the long-term orbital effects of such interaction on the Keplerian orbital elements of the moving particle. In Section 4 we look at the Sun–Mercury system and to PSR B1259-63 and PSR B1913+16 in order to see if the predicted effect could be measured. Section 5 is devoted to the conclusions.

2. The gravitomagnetic Stern–Gerlach force

In the context of the linearized gravitoelectromagnetism it turns out that a gravitomagnetic dipole moment for a gyroscope of spin \mathbf{s} is¹²

$$\vec{\mu}_g = -\frac{\mathbf{s}}{c} \quad (1)$$

and the energy of interaction with an external gravitomagnetic field \mathbf{B}_g is

$$H = -\vec{\mu}_g \cdot \mathbf{B}_g = \frac{\mathbf{s} \cdot \mathbf{B}_g}{c}. \quad (2)$$

Since, in general, the gravitomagnetic field is position-dependent, a Stern–Gerlach force arises from Equation (2)

$$\mathbf{F} = -\nabla H. \quad (3)$$

Let us consider a central spherical rotating body as source of the gravitomagnetic field, so that

$$\mathbf{B}_g = -\frac{GJ}{cr^3} \left[\hat{J} - 3 \left(\hat{J} \cdot \hat{r} \right) \hat{r} \right], \quad (4)$$

rotation of the Sun on the orbital angular momentum of the Earth–Moon system see reference⁷.

where G is the Newtonian gravitational constant, \hat{J} is the unit vector along the proper angular momentum of the central body and \hat{r} is the unit position vector. By assuming $\mathbf{J} = J\hat{z}$ in an inertial frame with the $\{x, y\}$ plane coinciding with the equatorial plane of the central mass, Equation (3) becomes

$$F_x = \frac{3GJ}{c^2 r^5} \left[s_x \left(\frac{5x^2 z}{r^2} - z \right) + s_y \left(\frac{5xyz}{r^2} \right) + s_z \left(\frac{5xz^2}{r^2} - x \right) \right], \quad (5)$$

$$F_y = \frac{3GJ}{c^2 r^5} \left[s_x \left(\frac{5xyz}{r^2} \right) + s_y \left(\frac{5y^2 z}{r^2} - z \right) + s_z \left(\frac{5yz^2}{r^2} - y \right) \right], \quad (6)$$

$$F_z = \frac{3GJ}{c^2 r^5} \left[s_x \left(\frac{5xz^2}{r^2} - x \right) + s_y \left(\frac{5yz^2}{r^2} - y \right) + s_z \left(\frac{5z^3}{r^2} - 3z \right) \right]. \quad (7)$$

Equation (5)- Equation (7) agree with the expression

$$\mathbf{F} = \frac{3GJ}{c^2 r^4} \left\{ \left[5(\mathbf{s} \cdot \hat{r})(\hat{J} \cdot \hat{r}) - \mathbf{s} \cdot \hat{J} \right] \hat{r} - (\mathbf{s} \cdot \hat{r})\hat{J} - (\hat{J} \cdot \hat{r})\mathbf{s} \right\} \quad (8)$$

of Equation (22) in reference¹² and with the first term of Equation (2) in reference¹.

3. The orbital effects

Now, let us work out the long-term orbital effects on the orbit of a spinning particle. We will consider it as an extended spherical body^b of radius l and (slowly) spinning with an angular velocity α , so that $s = \frac{2}{5}ml^2\alpha$. We will consider the gravitomagnetic spin-spin force of Equation (5)- Equation (7) as a small perturbation of the Keplerian monopole and we will adopt the Gauss perturbing equations

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} \left[Re \sin f + T \frac{p}{r} \right], \quad (9)$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na} \left\{ R \sin f + T \left[\cos f + \frac{1}{e} \left(1 - \frac{r}{a} \right) \right] \right\}, \quad (10)$$

$$\frac{di}{dt} = \frac{1}{na\sqrt{1-e^2}} N \frac{r}{a} \cos(\omega + f), \quad (11)$$

$$\frac{d\Omega}{dt} = \frac{1}{na \sin i \sqrt{1-e^2}} N \frac{r}{a} \sin(\omega + f), \quad (12)$$

$$\frac{d\omega}{dt} = -\cos i \frac{d\Omega}{dt} + \frac{\sqrt{1-e^2}}{nae} \left[-R \cos f + T \left(1 + \frac{r}{p} \right) \sin f \right], \quad (13)$$

$$\frac{d\mathcal{M}}{dt} = n - \frac{2}{na} R \frac{r}{a} - \sqrt{1-e^2} \left(\frac{d\omega}{dt} + \cos i \frac{d\Omega}{dt} \right), \quad (14)$$

where R , T , N are the projections of the perturbing acceleration onto the radial \hat{r} , transverse \hat{t} and out-of-plane \hat{n} directions of an orthonormal frame comoving with

^bIt should be noticed that, according to the authors of reference¹⁴, the straightforward extension of the spin-spin gravitational interaction for elementary particles to macroscopic extended rotating bodies should not be justified. Then, they follow a phenomenological approach not based a priori on General Relativity.

the orbiter, a , e , i , Ω , ω and \mathcal{M} are the orbiter's semimajor axis, eccentricity, inclination, longitude of the ascending node, argument of perigee and mean anomaly, respectively. Moreover, $p = a(1 - e^2)$, f is the true anomaly and $n = \sqrt{GMa^{-3}}$ is the Keplerian mean motion.

Now we will derive the components R , T , N from Equation (5)- Equation (7). In order to simplify the problem we will assume to consider only equatorial orbits. Moreover, we will further assume that the proper angular momentums of the two body are aligned, i.e. $\mathbf{s} = s\hat{k}$ as well. This is a situation quite common, e.g., with the planets and their moons in the Solar System to which our attention will be drawn. Indeed, even if of order $\mathcal{O}(c^{-2})$, the investigated effects are very small due to the dependence on r^{-4} , as can be noticed from Equation (5)- Equation (7) and by considering that x , y , z are all proportional to r ; so, only astronomical bodies could yield a chance for measuring them.

Let us make the x axis coincide with the line of the nodes: in this case

$$\hat{x} = \cos(\omega + f)\hat{r} + \cos i \sin(\omega + f)\hat{t} + \sin i \sin(\omega + f)\hat{n}, \quad (15)$$

$$\hat{y} = -\sin(\omega + f)\hat{r} + \cos i \cos(\omega + f)\hat{t} + \sin i \cos(\omega + f)\hat{n}, \quad (16)$$

$$\hat{z} = -\sin i \hat{t} + \cos i \hat{n}, \quad (17)$$

and

$$x = r \cos(\omega + f), \quad (18)$$

$$y = r \cos i \sin(\omega + f), \quad (19)$$

$$z = r \sin i \sin(\omega + f). \quad (20)$$

With Equation (16)- Equation (20), Equation (5)- Equation (7) become, in the case of equatorial orbits

$$R = -\frac{3GJ\bar{s}}{c^2 r^4} \cos[2(\omega + f)], \quad (21)$$

$$T = -\frac{3GJ\bar{s}}{c^2 r^4} \sin[2(\omega + f)], \quad (22)$$

$$N = 0 \quad (23)$$

where $\bar{s} = \frac{s}{m} = \frac{2}{5}l^2\alpha$. From Equation (23) and Equation (11)- Equation (12) it can be noticed that the inclination and the node, which, on the other hand, is not defined for equatorial orbits, are not affected by the gravitomagnetic spin-spin force.

By calculating Equation (21)- Equation (23) on the Keplerian unperturbed orbit

$$r_{\text{Kep}} = \frac{a(1 - e^2)}{1 + e \cos f}, \quad (24)$$

neglecting the terms of order $\mathcal{O}(e^2)$, inserting the obtained result into Equation (9)-

Equation (14) and averaging over an orbital revolution^c by means of

$$dt = \frac{(1 - e^2)^{\frac{3}{2}}}{n(1 + e \cos f)^2} df, \quad (25)$$

we obtain that only the eccentricity, the perigee and the mean anomaly experience long-term, harmonic perturbations with half the period of the perigee.

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{3GJ\bar{s}e}{4c^2a^5(1 - e^2)^2} \sin 2\omega, \quad (26)$$

$$\left\langle \frac{d\omega}{dt} \right\rangle = -\frac{15GJ\bar{s}}{4c^2a^4(1 - e^2)^2} \cos 2\omega, \quad (27)$$

$$\left\langle \frac{d\mathcal{M}}{dt} \right\rangle = \frac{15GJ\bar{s}}{4c^2a^4(1 - e^2)^{\frac{3}{2}}} \cos 2\omega. \quad (28)$$

4. Is it possible to measure the gravitomagnetic spin–spin force?

4.1. A Solar System scenario

In order to measure the small effects predicted by Equation (26)- Equation (27) the most suitable system seems to be the Sun–Mercury one. In Table 1 we quote the relevant orbital parameters of it. For them we have used the reference¹⁵. From

Table 1. Orbital parameters for the Sun and Mercury

Parameter	Description	Value	Units
M_{\odot}	Mass of the Sun	1.9891×10^{33}	g
R_{\odot}	Mean equatorial radius of the Sun	6.96×10^{10}	cm
ε_{\odot}	Tilt of the Sun's spin axis to the ecliptic	7.25	deg
T_{\odot}	Sidereal rotational period of the Sun	24.65	days
A.U.	Astronomical Unit	$1.4959787066 \times 10^{13}$	cm
M_m	Mass of Mercury	3.302×10^{26}	g
R_m	Mean equatorial radius of Mercury	2.4398×10^8	cm
a_m	Semimajor axis of Mercury	0.3870	A.U.
e_m	Eccentricity of Mercury	0.2505	-
i_m	Inclination of Mercury's orbit to the ecliptic	7.0048	deg
ε_m	Tilt of Mercury's spin axis to the ecliptic	0.01	deg
T_m	Sidereal rotational period of Mercury	58.646	days
P_m	Orbital period of Mercury	0.2408	years
G	Newtonian constant of gravitation	6.67259×10^{-8}	$\text{g}^{-1} \text{cm}^3 \text{s}^{-2}$
c	Speed of light in vacuum	2.9979×10^{10}	cm s^{-1}

Table 1 it can be inferred that, if we choose the ecliptic plane as $\{x, y\}$ plane, the assumptions made by us are satisfied rather well for the Sun and Mercury. Indeed, the Sun's spin axis is only slightly tilted to the vertical with respect to the ecliptic

^cIn the averaging process we consider the vector \bar{s} as constant, i.e. we neglect the De Sitter and Lense–Thirring precessions of the spin of the orbiting particle.

plane, the inclination of Mercury’s orbit to it amounts to few degrees and the spin axis of Mercury is vertical, so that it can be considered aligned to that of the Sun, at least to an approximate extent. The values of Table 1 yield for the Mercury’s perihelion shift

$$\left\langle \frac{d\omega}{dt} \right\rangle_{\text{Mercury}} = -(7.6 \times 10^{-10} \text{ mas/y}) \times \sin 2\omega. \quad (29)$$

Unfortunately, this is a value which is completely undetectable.

4.2. A pulsar scenario

Let us try to see if the predicted effects could be measured in an astrophysical context. The binary systems in which at least one member is a pulsar neutron star seem to be the optimal choice due to the large values of the eccentricity of the orbits of a not negligible fraction of them (70 out of more than 1300 known, i.e. about 5%). Moreover, the rapid rotations of the pulsars and the relatively small separations between the members of such systems could be helpful.

As can be inferred from Table 2, obtained from the data^d in reference¹⁷, the PSR B1259-63 system seems to be a good choice (See, e.g., <http://www.jb.man.ac.uk/~pulsar/> for a quick outlook of the discovered pulsars and their relevant orbital parameters). Of course, in that case our calculations should be considered as very preliminary order-of-magnitude estimates, just to understand if looking at pulsars more thoroughly and carefully could be fruitfully or not. In the case of PSR B1259-

Table 2. Orbital parameters for the PSR B1259-63/SS 2883 binary system

Parameter	Description	Value	Units
M_c	Mass of the companion	10	M_\odot
R_c	Equatorial radius of the companion	6	R_\odot
v_{max}	Break-up velocity of the companion	480	km s^{-1}
m	Mass of PSR B1259-63	1.4	M_\odot
r	Radius of PSR B1259-63	10	km
T	Period of rotation of PSR B1259-63	47.7620537	ms
P	Period of revolution of PSR B1259-63	1,236.7238	days
i	Inclination of PSR B1259-63	36 (144)	deg
e	Eccentricity of PSR B1259-63	0.86990	–
$x = \frac{a \sin i}{c}$	Projected semimajor axis of PSR B1259-63	1296.4	s

63 the companion is SS 2883, a 10th-magnitude star of spectral class B2e, almost 10 times more massive than the pulsar which orbits it in 3.4 years along a very eccentric path. In this case we will assume as $\{x, y\}$ reference plane the plane of sky which is a fixed plane normal to the line-of-sight. For the definition of the

^dThe value of the pulsar radius r is an estimate based on the first calculations by Oppenheimer and Volkoff¹⁶.

various orbital parameters of the pulsar systems see reference¹⁸. It should be noticed that in applying our results to PSR B1259-63 the following *caveat* hold. Neither the orbit of the pulsar can be considered equatorial^e, nor the spin \mathbf{J} of SS 2883 is vertical¹⁷. Moreover, we can calculate s for the pulsar, but we cannot say anything about its direction. On other hand, the value can be inferred for J is plausible because the stars of the same type of SS 2883 are well known.

By using the values of Table 2 we obtain for the periastron advance

$$\left\langle \frac{d\omega}{dt} \right\rangle_{\text{PSR B1259--63}} = -(3.3 \times 10^{-13} \text{ deg/y}) \times \sin 2\omega. \quad (30)$$

Notice that the period of the secular precession of the perigee of PSR B1259-63 amounts to 1.9×10^6 years^f, so that over reasonable observational time spans of few years the predicted harmonics would be similar to secular trends. According to Table 1 of reference¹⁷, the experimental sensitivity amounts to 10^{-6} deg/y for $\langle \dot{\omega} \rangle$, so that also in this case the gravitomagnetic spin-spin interaction turns out to be too small to be detected.

A more favorable situation occurs for the well known binary pulsar system of PSR B1913+16 in which the companion of the pulsar is probably another neutron star. The relevant orbital parameters are in Table 3 and have been taken from reference¹⁹. In this case we do not know anything about the period of rotation of

Table 3. Orbital parameters for the PSR B1913+16 binary system

Parameter	Description	Value	Units
m_c	Mass of the companion	1.3873	M_\odot ,
m	Mass of PSR B1913+16	1.4411	M_\odot ,
r	Radius of PSR B1913+16	10	km
T	Period of rotation of PSR B1913+16	59.029997929883	ms
P	Period of revolution of PSR B1913+16	$3.22997462736 \times 10^{-1}$	days
i	Inclination of PSR B1913+16	47 (133)	deg
e	Eccentricity of PSR B1913+16	0.6171309	—
$x = \frac{a \sin i}{c}$	Projected semimajor axis of PSR B1913+16	2.341759	s

the companion because it neither manifests to us in the visible nor in the radio regions of electromagnetic spectrum. We will assume for it 10^{-2} s, which is rather reasonable since the periods of the known pulsars range from 1.56×10^{-3} s to 6×10^{-1} s. An-order-of-magnitude calculation^g with our formulas yields

$$\left\langle \frac{d\omega}{dt} \right\rangle_{\text{PSR B1913+16}} = -(4.6 \times 10^{-7} \text{ deg/y}) \times \sin 2\omega. \quad (31)$$

^eNotice also that from pulsar orbit data reductions the inclination i cannot be determined unambiguously.

^fIt is mainly due to the classical effects of the oblateness of SS 2883; the relativistic gravitoelectric Einstein's precession amounts to only 3×10^{-5} deg/y.

^gThe spin of PSR B1913+16 may be affected by the relativistic geodetic precession⁵, but since its period would amount to 360 years it can be considered constant over an orbital revolution.

Notice that the period of the secular precession of the perigee of PSR B1913+16 amounts to 90 years^h, so that over reasonable observational time spans of few years the predicted harmonics would be similar to secular trends. Since for PSR B1913+16 the experimental accuracy in measuring $\langle\dot{\omega}\rangle$ amounts to about 1×10^{-5} deg/y (see reference²⁰), in this case we are not too far from the possibility of detecting the predicted effect.

5. Conclusions

In this paper, in the linearized approximation of gravitoelectromagnetism and at order $\mathcal{O}(c^{-2})$, we have calculated the influence of the spin \mathbf{s} of a particle on its geodesic orbital motion in an external gravitomagnetic field generated by a rotating body with spin \mathbf{J} . It has been assumed that the orbit lies in the equatorial plane of the central object and that the spins are both vertical to it. The orbital-averaged, long-term effects on the Keplerian orbital elements of the orbiter have been calculated by neglecting all terms of order $\mathcal{O}(e^2)$. It turns out that the eccentricity, the pericenter and the mean anomaly are affected by such spin-spin gravitomagnetic interaction by means of harmonic perturbations with half the period of the pericenter. In order to see if they are measurable, we have examined two possible astronomical scenarios: the Sun–Mercury system and a pair of binary millisecond pulsar systems. It turns out that they are far too small. Only for the pulsar PSR1913+16 the predicted periastron rate is not too far from the present experimental sensitivity. The possible discovery of new, highly eccentric and close binary pulsar systems, together with notable improvements of the accuracy in measuring the periastron rates could give some hopes to detect such tiny effects in future.

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^hIt is entirely due to the gravitoelectric Einstein’s precession.

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